

### “Triangle in a semi-circle: area vs altitude”

#### General

For a triangle fixed inside a semicircle, how does the area vary as the height changes, as you move across the diameter?

What does the graph of points look like?

#### Student Specific

Use the ‘Thought-Sheet’ (page 4) to guide you through how to consider the dynamics of this situation, and what sort of relationship it reveals. This task is best done by thinking about it first and then doing it second, in order to see how well formed your thoughts were.

#### Teacher Specific

The students can either construct this themselves (refer to page 2 of these notes, or HG055’s Construction Movie for the general process of how to transfer numerical measurements from a diagram to a set of axes within CabriJr), or use a pre-made file. There are obvious benefits to the students’ underlying appreciation of the problem if they do construct it themselves.

#### Acknowledgements

Original source of the idea for this interactive geometry construction came from Mr Eugene Olmstead, Elmira Free Academy (retired), Dundee, New York State, USA.

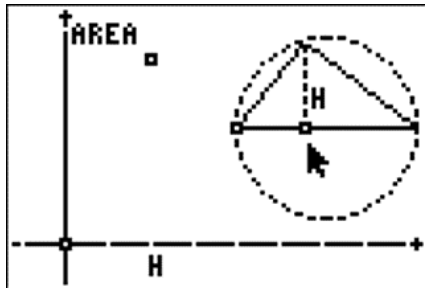
#### Feedback

Do you have any comments on these notes?

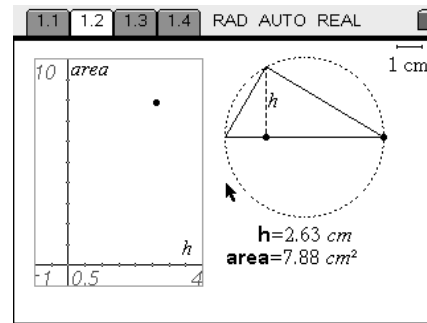
Please get in Contact via the website and help improve them further.

All contributors are acknowledged.

## Construction Suggestions for HG066



CabriJr Construction



TI-Nspire Construction

### Construction Suggestion for CabriJr .8xv file:

F5 > Hide/Show > Axes

Grab and drag origin to bottom left of screen.

Construct a segment, mark on its midpoint and draw a circle using this midpoint as its centre and the segment as its diameter.

Point a point on the diameter, then construct a line perpendicular to the diameter through that point.

Construct a triangle through the ends of the diameter and the intersection of the circle and the perpendicular line just drawn.

Measure the height's length and the triangle's area.

F3 > Compass to capture the length of the height and place the generated circle with its centre on the origin.

Draw in perpendicular to x-axis through intersection point of this circle with the x-axis.

F5 > Hide/Show > Object and hide this circle

Draw a new, short segment on the y-axis, upwards from the origin.

Use the dilation command to enlarge this segment by the numerical value of the area.

Adjust the initial short segment's length, so that the enlarged image segment does not exceed the height of the y axis.

Draw in a perpendicular to the y-axis through the end point of the image segment.

Mark the point of intersection of the two perpendicular lines, and then hide the lines, including the original vertical segment.

### Construction Suggestion for TI-Nspire .tns file:

Page Add Graphs & Geometry

1.2 Menu > View > Plane Geometry View

Menu > View > Show Analytic Window

Ctrl + G to Hide Entry Line

Resize top and bottom edges of the Analytic Window to the full height of the screen.

Construct a segment, mark on its midpoint and draw a circle using this midpoint as its centre and the segment as its diameter.

Point a point on the diameter, then construct a line perpendicular to the diameter through that point.

Construct a triangle through the ends of the diameter and the intersection of the circle and the perpendicular line just drawn.

Measure the height's length and the triangle's area.

Move the cursor over each numerical measure and press Var > Store Var

Type in names of Vars to be h and area, one at a time.

Menu > Construction > Measurement Transfer, then click on a Var, then click on an axis.

Construct Perpendiculars to axes through the respective Measurement Transferred points.

Plot point at intersection of perpendiculars.

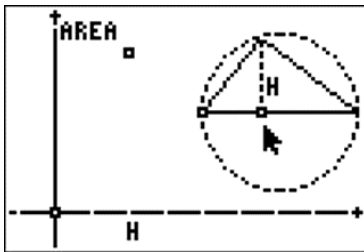
Hide appropriate items by Ctrl > Menu

- Page 1.3 Label columns with h.c and area.c  
[the “.c” bit is a chosen code to mean “the captured value of the variable”]  
Type in capture(h,1)  
Type in capture(area,1)  
[the parameter of “1” means capture automatically. Use “0” for manual capture]
- Page 1.4 Click on each axis and select from the variables that end in “.c”  
After some data has been captured, Menu > Window/Zoom > Zoom-Data to automatically scale axes.

Name: \_\_\_\_\_

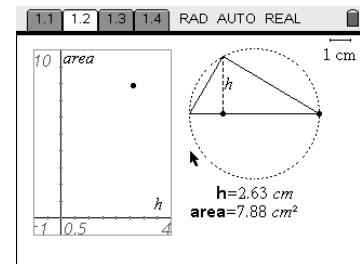
Date: \_\_\_\_\_

## Connections – HG066 – Thought Sheet



For a triangle fixed inside a semicircle, how does the area vary as the height changes, as you move across the diameter?

What does the graph of points look like?



For the geometrical construction detailed above, jot down your answers to the following key questions:

1. Is the data relationship linear, or non-linear?
  - a) If it's linear, what is the slope of the line?
  - b) If it's non-linear, is the curve upwards or downwards?
2. Is there a direct, or indirect variation?  
\_\_\_\_\_
3. Is the data relationship symmetrical in any way?  
\_\_\_\_\_
4. Is there a maximum, or a minimum?  
\_\_\_\_\_
5. Are there places where it cuts the x-axis (called 'zeros') and, if so, what do they represent?  
\_\_\_\_\_
6. Does it cut the y-axis and, if so, what does that point represent?  
\_\_\_\_\_
7. Can you now draw the shape of the curve?  
Sketch on it the axes on one of the screenshots at the top of this sheet.
8. Can you now construct this interactive geometry file, or use the one provided to you, to check all of your thoughts so far?
9. Can you use algebra to work out the equation of the curve?

If you are using a TI-Nspire, you can check how good your answer is by plotting your resultant function on Page 1.2's axes. Alternatively, you can fit a regression curve to the captured data that will be displayed on Page 1.4